Assignment 6

1. Find approximations for the two roots of the polynomial $0.0002358x^2 - 5535.0x + 0.00003513$ using the quadratic formula you learned in secondary school. Then, find the same roots, but choosing the appropriate formula for each.

2. Find approximations for the two roots of the polynomial $0.0002358x^2 + 5535.0x - 0.00003513$ using the quadratic formula you learned in secondary school. Then, find the same roots, but choosing the appropriate formula for each.

3. Given the function $f(x) = x^3 - x^2 - x - 1$, approximate the real root using two steps of each of:

a. Newton's method starting with $x_0 = 2.0$,
b. the bisection method starting with [1, 2],
c. the bracketed secant method starting with [1, 2] (optional), and
d. the secant method starting with $x_0 = 2.0$ and $x_1 = 1.9$.

4. Given the same function as in Question 3, approximate the first positive root using one step of each of:

a. Muller's method starting with $x_0 = 2.0$, $x_1 = 1.9$ and $x_2 = 1.8$ (optional), and
b. inverse quadratic interpolation with the same three points.

5. Given the function $f(x) = x^2\cos(0.4x)e^{-0.3x}$, approximate the first positive root using two steps of each of:

a. Newton's method starting with $x_0 = 4.0$,
b. the bisection method starting with [3, 4],
c. the bracketed secant method starting with [3, 4] (optional), and
d. the secant method starting with $x_0 = 3.8$ and $x_1 = 3.9$.

6. Given the same function as in Question 5, approximate the first positive root using one step of each of:

a. Muller's method starting with $x_0 = 3.8$, $x_1 = 4.0$ and $x_2 = 3.9$ (optional), and
b. inverse quadratic interpolation with the same three points.

7. Apply two steps of the Jacobi method or the Gauss-Seidel method (optional) and then two steps of successive over-relaxation applied to these with $\omega = 0.97$ for the Jacobi method and $\omega = 1.03$ for the Gauss-Seidel method to find an approximation of the solution to:

$$\begin{pmatrix} 10 & 2 \\ 2 & 10 \end{pmatrix}\mathbf{u} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}.$$

8. Apply two steps of the Jacobi method or the Gauss-Seidel method (optional) and then two steps of successive over-relaxation applied to these with $\omega = 0.99$ for the Jacobi method and $\omega = 1.08$ for the Gauss-Seidel method to find an approximation of the solution to:

$$\begin{pmatrix} 5 & 2 & 1 \\ 2 & 10 & -3 \\ 1 & -3 & 20 \end{pmatrix}\mathbf{u} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}.$$

9. The following system is given with the solution:

$$\begin{pmatrix} 2 & 5 \\ 5 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 7 \\ 6 \end{pmatrix}.$$

If you were to try to apply the Jacobi method or the Gauss-Seidel method to find the solution, does it seem to converge? Why does this happen?

10. Could you use the method of successive over-relaxation with a method such as Newton's method? For example, if you found that $x_1 > x_2 > x_3 > x_4$, might it not make sense to try to use $\omega = 1.05$? Similarly, if successive approximations move back and forth, might it not make sense to try to use $\omega = 0.95$?